Chaos theory & algorithmic compositions

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1 Introduction

This paper dips its toes into the deep subject that is algorithmic composition with chaotic systems. I extracted the explanation on chaos from various sources. Methods of connecting it to algorithmic composition stem mostly from my own experimentations in the subject with great influences from my teachers M. IJzerman, M. Wierckx, M. Groenewegen & G. van Wolferen. Who have given me valuable insights on the subject of algorithmic composition during multiple projects for System Design for Sound and Music(S.O.G.M.).

2 What is chaos

In order to talk clearly about chaos theory let's start off by disconnecting the meaning of the word chaos from its traditional meaning: *"Complete disorder and confusion"*(*Oxford Dictionary of English* 2012). Chaos is not random but deterministic. If you run a formula a hundred times with the same parameters, you get the same output every time. Chaos theory is a part of a math subject called dynamics, which deals with change and systems that evolve in time. The way a chaotic system constantly evolves, is by feeding back the output of the first iteration into the second, creating a non-linear feedback system. Although the output of chaos may appear random, if observed from a distance, patterns begin to emerge.(*Chaos theory* 2016; White 2009; Boeing 2009; Strogatz 1994)

3 Chaos and music

What reason could we have to string these two distant worlds together, music and math? If we take a look at how music is structured, we will notice some resemblance with the chaos. "Musical development or variation can be viewed as the transformation or distortion of a simple entity (a motive), often followed by some sort of return to the original motive."(Pressing 1988) The task of a music composer is to construct patterns, out of smaller sub-components. Melodic and textural components are bound into patterns by rhythmic components. Who are then strung together into new patterns, are varied upon, or contrasted by whole other patterns. The piece is complete when the balance between variation, repetition and new is achieved. Which depends on the desired style of music, and the composer. So, music in its essence deals with change, it's a system that evolves over time (dynamics). And is created by reflecting of what is already there (feedback). The same characteristics of a chaotic system. I'm not saying that chaos will work as a drop in replacement for music, but these parallels make it a good source of musical inspirations, or a tool to build whole new compositions.

4 Methodology™

Just like when composing with pen and paper, algorithmic systems won't give you access to a big red button that automatically spews out a hit when pressed. Results are a matter of continuous experimentation and sometimes a dash of luck. There is also no need to exclude algorithmic and traditional approach to composition from each other. Tweaking until it sounds good, with whatever techniques available is the name of the game.

4.1 The formula

Chaos theory is a wast subject, so there are plenty of formulas and variations thereof to choose from. So head to the nearest google search bar, or a library with a math section, and experiment with what you find. The formula will determine how much variation is available between generated patterns, and how many values you can extract from a single iteration. A good way to examine the patterns a chaotic system produces, is to plot the output and look at the shapes that occur. Lets look at an example of a simple chaotic formula, known as the logistic map.

$$\dot{x} = rx(1 - x)$$

Sometimes used as a simple model of population growth with finite resources (Boeing 2009). It has one variable x, which is here used to plot the bifurcation diagram in *fig.1*.

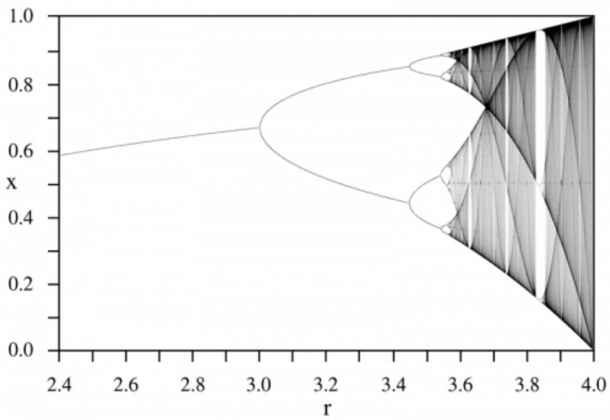

Figure 1: Logistic map (White 2009)

As you can see there is not much excitement going on in *fig.1* in terms of patterns. The output first settles to an oscillation between few points, with a steep transition into chaotic oscillation. This could be utilised to add variation to a control signal, or to arpeggiate a chord, but is a bit monotonic if we want the end result to be a melody. Let's take a look at a formula with a more diverse output.

$$sgn(z) = \frac{z}{abs(z)}$$
$$\dot{x} = y - sgn(z) * \sqrt{abs(b * x - o)}$$
$$\dot{y} = t - x$$

Now we're talking! This one, called hopalong orbits, has two variables, that are fed back into the formula ($x \& y$), and has additional four settable variables (z, t, b, o). The possible patterns you can extract from it is enormous. Take a look at some examples in *fig.2*.

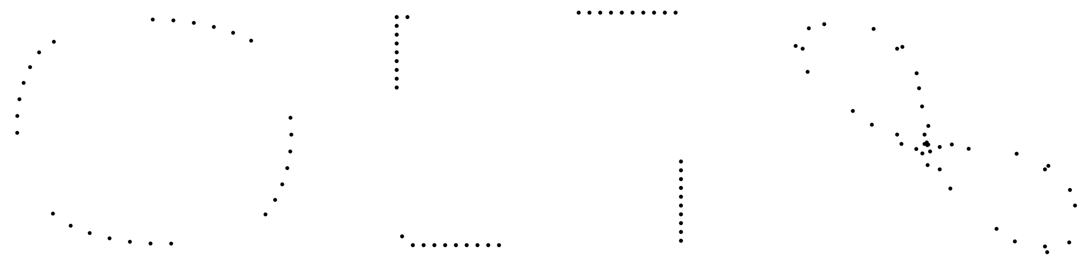

Figure 2: Patterns generated with hopalong

4.2 The number stream

Having chosen a formula we can get cracking and observe the numbers that come rolling out it. If we sonify the logistic map (e.g. by mapping variable x, to a two octave note range). The connection between what we see and hear is clear. However in the case of the hopalong formula the patterns are visually easy to see. But playing back the number stream won't result in a $1:1$ connection with what we see. Not only does it become more abstract because we are now translating a two dimensional pattern to one dimension. But the order in which the formula makes the dots, is not a gradual progress of putting down a dot, moving to the closest one, and put a dot there like a pointillist artist might do. It puts down a point, jumps away along an invisible orbit and puts another point down (hence the name hop a long orbits), and on it goes until a pattern emerges (as seen in *fig.3*). Playing it back we will hear a pattern that while jumping in circles, results in a shape, instead of gradually tracing the contours of it as you might have expected.

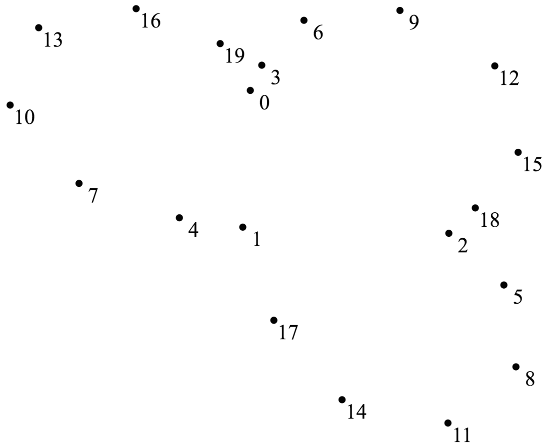

Figure 3: A hopalong pattern. Points numbered by their order in the stream.

4.3 Controlling the stream

Depending on how the formula you picked behaves and what characteristic of it you want to accentuate, it may be handy to buffer the output of the chaos, instead of using the stream directly. By filling an array with our chosen number of iterations it becomes easy to sort and map it content to values. In case you are working with the hopalong formula, you could order the points in the buffer by distance from each other so their order follows the contours of the pattern, opposed to what was discussed in section 4.2.

Let's not overlook the biggest factor in the contents of the stream: the initial conditions used to generate data. If we think of a set of parameters as a phrase, you get a variation on it, by making a tiny adjustment to the parameters. So if a small change is more likely to generate a variation of the previous state, and as the change gets bigger, the possibility of the output being completely different rises. We can use parameter variations to implement a pseudo ABA structure in the composition.

Due to the nature of chaos obtaining structure in this way won't be bulletproof, a tiny change in the initial conditions could lead to a totally different output. You could enforce the structure by comparing the streams you wish to vary upon, and the stream that is supposedly the variations. We could convert each stream into a number set and calculate the Jaccard index(*Jaccard index* 2016; Teknomo 2015). The result is a value between 0 and 1, indicating how alike the two streams are. So two streams with a Jaccard index of 1, are the same, and as they approach 0, become more different. It's worth noting since the Jaccard index is calculated using a number set, the order of elements and whether it contains duplicates is not taken into account.(*Set* 2016)

An alternative to running the formula with different parameters is to continue iterating with the same parameters. Our initial pattern could than be x many iterations of the hopalong formula, and to get a variations on that pattern, we start by letting the formula iterate n many times before we start putting the stream into the buffer. The effect of this can be seen in *fig.4*. In this case also a sort of back and forth ABA structure appears in the changes of the shape and as you travel further with each iteration, it gets more and more likely that the shapes strays away from the back and forth motion.

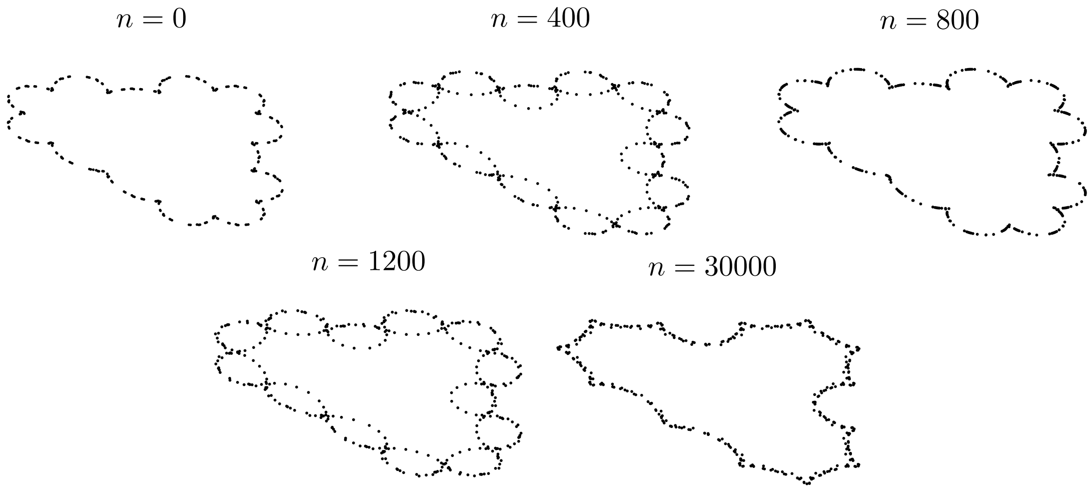

Figure 4: 400 points with the same initial conditions but a offset of n iterations.

4.4 Map all the things!

Now in order to hear some music we just have to assign the variables to musical parameters. Let's say we have picked the hopalong formula which outputs two values, x & y. We could map x to pitch, and y to duration. Or to make a chaotic counterpoint we can run two instances of hopalong (with different parameters) side by side, and use one formula for pitches and the other for the durations. Another possibility is to run two formulas side by side, with the same parameters and instead using an offset in the iterations to create a counter point.

How you want to map values is also open to experimentation. Instead of just mapping one axis to one value, we could assign a value to a point by its distance from the middle of the plot. Or we could spread the values we want to map to on a grid, and if a point is within a box in the grid, that element gets chosen. And we can then take it even further by combining multiple mappings, as in assigning a points distance from the middle to pitch, x to rhythm and y to velocity.

There is no need to constrain ourselves to only pitch and duration, we have dynamics, texture, scales, and if we zoom out: structure, instrumentations and build ups. We could take a note from composer Diana Dabby who among other things used chaos to rearrange already finished composition (Johnson 2013).

5 Composition

Although we are at the end of this paper, there are still countless methods and possibilities to explore. There are multiple chaotic systems, although not discussed in this paper, that have applications in a musical context e.g. strange attractors, fractals and double pendulums(*Chaos theory* 2016). You as the composer get to make the decisions how the composition will come together. Make use of a single chaotic formula for inspiration, or connect several of them together into an integrate autonomous music composition system. It's time to get chaotic!

References

[1] Geoff Boeing. *Chaos Theory and the Logistic Map*. online. 2009. URL: `http://geoffboeing.com/2015/03/chaos-theory-logistic-map/`.

[2] *Chaos theory*. wikipedia. 2016. URL: `www.en.wikipedia.org/wiki/Chaos_theory`.

[3] *Jaccard index*. wikipedia. 2016. URL: `https://en.wikipedia.org/wiki/Jaccard_index`.

[4] Carolyn Y. Johnson. *What a little chaos does for music*. 2013. URL: `https://www.bostonglobe.com/ideas/2013/06/15/what-little-chaos-does-for-music/QLkNTkPIgmec20Db39oHbN/story.html`.

[5] *Oxford Dictionary of English*. 3rd Edition. Oxford University Press, 2012.

[6] Jeff Pressing. "Nonlinear maps as generators of musical design". In: *Computer Music Journal* 12(2) (1988), pp. 35–46.

[7] *Set*. wikipedia. 2016. URL: `https://en.wikipedia.org/wiki/Set_(mathematics)`.

[8] S.H. Strogatz. *Nonlinear Dynamics and Chaos: With Applications to Physics, Biology, Chemistry, and Engineering*. Perseus books, 1994.

[9] Kardi Teknomo. *Similarity Measurement*. online. 2015. URL: `http://people.revoledu.com/kardi/tutorial/Similarity/Jaccard.html`.

[10] Joey White. *Chaos and the logistic map*. online. 2009. URL: `http://hephaestusaudio.com/delphi/2009/02/16/chaos-and-the-logistic-map/`.